Applications of Painlevé Functions to Nonlinear Wave Equations

Robert Buckingham
Department of Mathematical Sciences, University of Cincinnati, Ohio 45221, U.S.A.
Email: buckinrt@uc.edu

Abstract:

The solution to a nonlinear wave equation with given Cauchy data often displays two or more qualitatively different behaviors in different space-time regions, such as having oscillatory and non-oscillatory zones. The boundaries between these regions may become well-defined in certain limits (such as long time or small dispersion), making it natural to consider the transition behavior between the two regions. It has recently been discovered that certain transition regions for solutions of nonlinear wave equations (such as KdV, focusing NLS, and Camassa-Holm) can be universally described for wide classes of initial conditions in terms of Painlevé functions [1,2,4]. These functions, which are solutions of nonlinear ordinary differential equations, play a role for nonlinear equations analagous to the role played by the classical special functions for linear equations. We will present our recent result with P. Miller [3] establishing Painlevé-type asymptotics in solutions of the semiclassical sine-Gordon equation.

References:

1. M. Bertola and A. Tovbis, Universality for the focusing nonlinear Schrödinger equation at the gradient catastrophe point: Rational breathers and poles of the tritronquée solution to Painlevé I, to appear in *Comm. Pure Appl. Math.*, arXiv:1004.1828 (2010).
2. A. Boutet de Monvel, A. Its, and D. Shepelsky, Painlevé-type asymptotics for the Camassa-Holm equation, *SIAM J. Math. Anal.* **42** (2010), 1854–1873.
3. R. Buckingham and P. Miller, The sine-Gordon equation in the semiclassical limit: critical behavior near a separatrix, *J. Anal. Math* **118** (2012), 397–492.
4. T. Claeys and T. Grava, Painlevé II asymptotics near the leading edge of the oscillatory zone for the Kortewegde Vries equation in the smalldispersion limit, *Comm. Pure Appl. Math.* **63** (2010), 203–232.